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The Tower of Hanoi (1989)

In the traditional statement of the problem, there are three poles and n discs, something like barbell weights in that they can be stacked on a pole; their sizes increase from smallest to largest in strict monotonic order, and in the initial configuration they are stacked in a pyramid, smallest to largest, and the rule is that a disc can only sit on one larger than itself — then: move the stack of discs from one pole to another, with the third as an auxiliary storage device — an accumulator. — The recursion is then defined as follows: move the first (n - 1) discs to the accumulator; move the bottom disc to the target pole; move the first (n - 1) discs back on top of it.

For a single disc obviously one move suffices; for two, you move the top disc to the accumulator, the bottom disc from the first pole to the second, and the top disc from the accumulator to the second pole, a total of three moves; three discs take seven moves, and in general the algorithm requires 2^n - 1 moves to transfer n discs from the first pole to the second.

The question occurs to me, whether introducing less than exponentially many additional accumulator poles could reduce the time required to something polynomial in the number of discs. The answer is no; with the aid of a lemma that savors of the calculus of finite differences, I prove it......